1. ⓐ Write down the four Maxwell equations and describe their physical importance, seperately. (8%)

ⓑ Derive the wave equations for the vector potential $\vec{A}$ and the scalar potential Φ, from the Maxwell equations. The answer in tensor form is as follows

$$\square^2 A_\mu = -\frac{4\pi}{c} J_\mu .$$

Verify it. (8%)

Define four vectors

$$X_\mu = (\vec{X}, ict)$$
$$A_\mu = (\vec{A}, i\Phi)$$
$$J_\mu = (\vec{J}, ic\rho).$$

Lorentz condition $\frac{1}{c}\frac{\partial \Phi}{\partial t} + \vec{\nabla}\cdot\vec{A} = 0$

or in tensor form

$$\frac{\partial A_\mu}{\partial X_\mu} = 0$$

ⓒ Field strength tensor is defined as

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial X_\mu} - \frac{\partial A_\mu}{\partial X_\nu}$$

Fill out the following explicit form of the field strength tensor. (答案須寫於答題紙)

$$(F_{\mu\nu}) = \begin{pmatrix} 0 & B_3 & -B_2 & -iE_1 \\ \square & 0 & B_1 & -iE_2 \\ \square & \square & 0 & -iE_3 \\ \square & \square & \square & 0 \end{pmatrix}$$

(4%)

ⓓ Which Maxwell equation is reprented by the following equation

$$\frac{\partial F_{\mu\nu}}{\partial X_\lambda} + \frac{\partial F_{\lambda\mu}}{\partial X_\nu} + \frac{\partial F_{\nu\lambda}}{\partial X_\mu} = 0$$

2. Use the method of images to find the force between the point charge q and the grounded conducting sphere with radius of a. The point charge is located at $\vec{y}$ relative to the origin of the sphere.
Discuss your result. (25%)

3. A localized distribution of charge is described by the charge density $\rho(\vec{x}')$, which is nonvanishing only inside a sphere of radius R^* around some origin. The potential outside the sphere can be expressed in multipole expansion in the following form

$$\Phi(\vec{x}) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{4\pi}{2l+1} q_{lm} \frac{Y_{lm}(\theta,\phi)}{r^{l+1}}$$

where

$$q_{lm} = \int Y^*_{lm}(\theta',\phi')\, r'^{l} \rho(\vec{x}')\, d^3x'$$

Verify the following expressions

ⓐ $q_{00} = \frac{1}{\sqrt{4\pi}}\, q$ (5%)

ⓑ $q_{11} = -\sqrt{\frac{3}{8\pi}}\,(P_x - iP_y)$ (5%)

ⓒ $q_{10} = \sqrt{\frac{3}{4\pi}}\, P_z$ (5%)

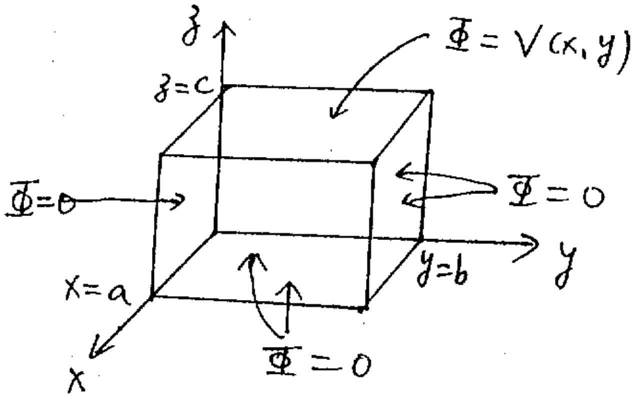
ⓓ and show that the $l=1$ multipole terms correspond to $\frac{\vec{P}\cdot\vec{x}}{r^3}$ obtained by direct Taylor series expansion of $\frac{1}{|\vec{x}-\vec{x}'|}$ in rectangular coordinates. (10%)

$$\left(\begin{aligned} Y_{00} &= \frac{1}{\sqrt{4\pi}} \\ Y_{11} &= -\sqrt{\frac{3}{8\pi}} \sin\theta\, e^{i\phi} \\ Y_{10} &= \sqrt{\frac{3}{4\pi}} \cos\theta \end{aligned} \right)$$

國立中山大學八十五學年度碩博士班招生考試試題

科 目：電動力學 ([illegible]研究所)

4. Find the potential $\Phi(x, y, z)$ of the following case



Hollow, rectangular box with five sides at zero potential, while the sixth ($z = c$) has the specified potential $\Phi = V(x, y)$

(25%)

物理研究所電磁學

1. Two long parallel wires separated at a distance, R, bear currents I_1 and I_2.

(a) If the currents flow in the same direction, is the force between the two wires attractive or repulsive ? (5 points)

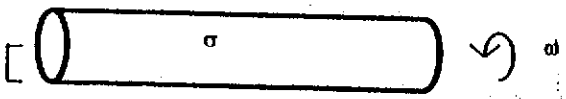
(b) What is the magnetic force **per unit length** on the wires ? (5 points)

2. A long cylindrical shell conductor of radius, R, carries surface charge density, σ.

(a) Find out the pressure exerted on the shell due to the repulsion of electric charge. (10 points)

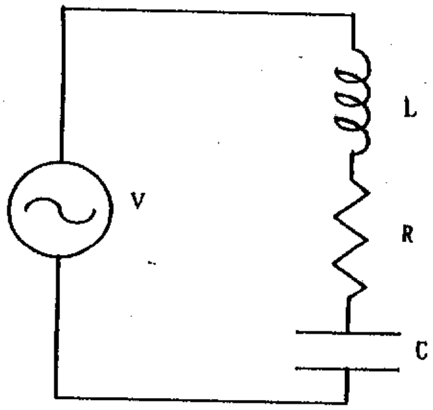
(b) If the cylinder is rotated with respect to its axis with angular velocity, ω, find out the now generated magnetic field. (10 points)

(c) Since the charge on the cylinder is now experiencing noninertial motion (i.e. rotation instead of translation), will there be any electromagnetic radiation generated ? Explain your answer. (5 points)



3. Show that the conservation of charge is built into the Maxwell's equations. (10 points)

4. A source of alternating voltage feeds a series LRC circuit, as shown in the figure.



Express and plot the impedance ratio $\frac{|Z|}{R}$ as a function of the AC driving frequency, ω.(10 points)

5. An amplified ultrafast pulse laser system is able to generate laser pulse of energy 1 joule/pulse with duration 100 femtoseconds ($1 fs = 10^{-15}$ sec). The laser beam can be focused down to a spot of 100 μm in diameter. Estimate the peak values of electric field, **E**, and magnetic field, **B**. (10 points)

6. (a) Explain the following terms, **total reflection, critical angle of incidence**, and **Brewster angle**.(10 points)

(b) What is the critical angle of incidence for water ? The index of refraction of water is approximately 1.33. (5 points)

7. List in order of increasing frequency of the following electromagnetic wave: infrared, green light, yellow light, telephone line, AC power line, blue light , X-ray, g -ray, microwave, AM radio wave, ultraviolet. (5 points)

8. A metallic spherical shell of radius 0.1 meter is charged to a potential of 10,000 volts relative to ground.

(a) Find the strength of electric field 1 meter away from the spherical center. (5 points)

(b) Find the potential at the center of the sphere.(5 points)

(c) What is the capacitance of the metallic sphere ?(5 points)

Vector definitions, identities, and theorems

Definitions

Rectangular coordinates

1. $\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$
2. $\nabla \cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$
3. $\nabla \times A = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{z}$
4. $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$
5. $\nabla^2 A = \nabla^2 A_x \hat{x} + \nabla^2 A_y \hat{y} + \nabla^2 A_z \hat{z} = \nabla(\nabla \cdot A) - \nabla \times (\nabla \times A)$

Cylindrical coordinates

6. $\nabla f = \frac{\partial f}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}$
7. $\nabla \cdot A = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$
8. $\nabla \times A = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{\rho} + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\hat{\phi} + \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho A_\phi) - \frac{\partial A_\rho}{\partial \phi}\right]\hat{z}$
9. $\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$
10. $\nabla^2 A = \nabla(\nabla \cdot A) - \nabla \times (\nabla \times A)$ (Sec. 1.11.6).

Spherical coordinates

11. $\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{\partial f}{r\,\partial \theta}\hat{\theta} + \frac{1}{r \sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$
12. $\nabla \cdot A = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r \sin\theta}\frac{\partial A_\phi}{\partial \phi}$
13. $\nabla \times A = \frac{1}{r \sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial (r A_\phi)}{\partial r}\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial (r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right]\hat{\phi}$
14. $\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$
15. $\nabla^2 A = \nabla(\nabla \cdot A) - \nabla \times \nabla \times A$ (Sec. 1.11.6).

Identities

1. $(A \times B) \cdot C = A \cdot (B \times C)$
2. $A \times (B \times C) = B(A \cdot C) - C(A \cdot B)$
3. $\nabla (fg) = f\nabla g + g\nabla f$
4. $\nabla (a/b) = (1/b)\nabla a - (a/b^2)\nabla b$
5. $\nabla (A \cdot B) = (B \cdot \nabla)A + (A \cdot \nabla)B + B \times (\nabla \times A) + A \times (\nabla \times B)$
6. $\nabla \cdot (fA) = (\nabla f) \cdot A + f(\nabla \cdot A)$
7. $\nabla \cdot (A \times B) = B \cdot (\nabla \times A) - A \cdot (\nabla \times B)$
8. $(\nabla \cdot \nabla) f = \nabla^2 f$
9. $\nabla \times (\nabla f) = 0$
10. $\nabla \cdot (\nabla \times A) = 0$
11. $\nabla \times (fA) = (\nabla f) \times A + f(\nabla \times A)$
12. $\nabla \times (A \times B) = (B \cdot \nabla)A - (A \cdot \nabla)B + (\nabla \cdot B)A - (\nabla \cdot A)B$
13. $\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$ (Sec. 1.11.6)
14. $(A \cdot \nabla)B = \left[A_x \frac{\partial B_x}{\partial x} + A_y \frac{\partial B_x}{\partial y} + A_z \frac{\partial B_x}{\partial z}\right]\hat{x} + \left[A_x \frac{\partial B_y}{\partial x} + A_y \frac{\partial B_y}{\partial y} + A_z \frac{\partial B_y}{\partial z}\right]\hat{y} + \left[A_x \frac{\partial B_z}{\partial x} + A_y \frac{\partial B_z}{\partial y} + A_z \frac{\partial B_z}{\partial z}\right]\hat{z}$
15. $\nabla'(1/r) = \hat{r}/r^2$. This is the gradient calculated at (x', y', z'), and r is the vector r pointing from (x', y', z') to (x, y, z).
16. $\nabla(1/r) = -\hat{r}/r^2$. This is the gradient calculated at (x, y, z) with the same vector r.
17. $\mathcal{A} = \frac{1}{2}\oint_C r \times dl$, where the surface of area $\mathcal{A}$ is plane. The vector r extends from an arbitrary origin to a point on the curve C that bounds $\mathcal{A}$.
18. $\int_v \nabla f\, dv = \int_{\mathcal{A}} f\, d\mathcal{A}$
19. $\int_v (\nabla \times A)\, dv = -\int_{\mathcal{A}} A \times d\mathcal{A}$, where $\mathcal{A}$ is the area of the closed surface that bounds the volume v.
20. $\oint_C f\, dl = -\int_{\mathcal{A}} \nabla f \times d\mathcal{A}$ where C is the closed curve that bounds the open surface of area $\mathcal{A}$.

Theorems

1. The divergence theorem. $\int_{\mathcal{A}} A \cdot d\mathcal{A} = \int_v \nabla \cdot A\, dv$ where $\mathcal{A}$ is the area of the closed surface that bounds the volume v.
2. Stokes's theorem: $\oint_C A \cdot dl = \int_{\mathcal{A}} (\nabla \times A) \cdot d\mathcal{A}$.

Physical constants†

Elementary charge	$e = 1.602177 \times 10^{-19}$ C
Electron rest mass	$m = 9.10938 \times 10^{-31}$ kg
	$= 5.10999 \times 10^5$ eV
Proton rest mass	$m_p = 1.67262 \times 10^{-27}$ kg
	$= 9.38272 \times 10^8$ eV
Speed of light in vacuum	$c = 2.99792458 \times 10^8$ m/s
Permittivity of vacuum	$\epsilon_0 = 8.85418782 \times 10^{-12}$ F/m
	$1/4\pi\epsilon_0 = 8.9875518 \times 10^9$ m/F
Permeability of vacuum	$\mu_0 = 4\pi \times 10^{-7}$ H/m
Avogadro constant	$N_A = 6.02213 \times 10^{23}$ mol^{-1}
Boltzmann constant	$k = 1.3806 \times 10^{-23}$ J/K
Planck constant	$h = 6.62607 \times 10^{-34}$ J · s
	$\hbar = h/(2\pi) = 1.054572 \times 10^{-34}$ J · s
Gravitational constant	$G = 6.672 \times 10^{-11}$ N · m^2/kg^2
Mass of the sun	1.98596×10^{30} kg
Radius of the sun	6.965×10^8 m
Mean sun-earth distance	1.495×10^{11} m
Earth's mean orbital speed	2.98×10^4 m/s
Mass of the earth	5.974×10^{24} kg
Radius of the earth	6.378×10^6 m
Mass of the moon	7.3305×10^{22} kg
Radius of the moon	1.74×10^6 m
Moon-earth distance	3.84393×10^8 m

† Codata Bulletin, November 1986

Maxwell's equations for stationary media

A. Differential form with E, B, P, M

$$\nabla \cdot E = \frac{\rho_f - \nabla \cdot P}{\epsilon_0}, \qquad \nabla \times E + \frac{\partial B}{\partial t} = 0,$$

$$\nabla \cdot B = 0, \qquad \nabla \times B - \frac{1}{c^2}\frac{\partial E}{\partial t} = \mu_0\left(J_f + \frac{\partial P}{\partial t} + \nabla \times M\right).$$

B. Integral form

$$\int_{\mathcal{A}} E \cdot d\mathcal{A} = \frac{1}{\epsilon_0}\int_v (\rho_f - \nabla \cdot P)\, dv$$

$$\int_{\mathcal{A}} B \cdot d\mathcal{A} = 0,$$

$$\int_{\mathcal{A}} (\nabla \times E) \cdot d\mathcal{A} = \oint_C E \cdot dl = -\frac{d\Lambda}{dt},$$

$$\int_{\mathcal{A}} (\nabla \times B) \cdot d\mathcal{A} = \oint_C B \cdot dl = \mu_0 \int_{\mathcal{A}} \left(J_f + \frac{\partial P}{\partial t} + \nabla \times M + \epsilon_0 \frac{\partial E}{\partial t}\right) \cdot d\mathcal{A}.$$

C. Differential form with E, D, H, B

$$\nabla \cdot D = \rho_f, \qquad \nabla \times E + \frac{\partial B}{\partial t} = 0,$$

$$\nabla \cdot B = 0, \qquad \nabla \times H - \frac{\partial D}{\partial t} = J_f.$$

D. Sinusoidal fields with E, D, H, B and for linear media

$$\nabla \cdot \epsilon E = \rho_f, \qquad \nabla \times E + j\omega\mu H = 0$$

$$\nabla \cdot \mu H = 0, \qquad \nabla \times H - j\omega\epsilon_r\epsilon_0 E = J_f.$$

Quantum Mechanics Ph.D. Entrance Exam

15%

1. We define
$$(\Delta A)^2 = < A^2 > - < A >^2 = < (A - < A >)^2 >$$
for the operator A of a physical observable. Let
$$U = A - < A > \ and\ V = B - < B >,$$
where A and B are the operators of any two physical observables. We consider a function defined by
$$\phi = U\psi + i\lambda V\psi,$$
where λ is a parameter and ψ is a state wavefunction. Using the positive definite property of
$$I(\lambda) = \int dx \phi^* \phi \geq 0,$$
show that
$$(\Delta A)^2 (\Delta B)^2 \geq \frac{1}{4} < i[A, B] >^2$$
and describe its physical meaning. Give as many examples as you can.

15%

2. If the potential of an electron in an atom is spherical and is given by $V(r) = -V_0$ for $r \leq R$ and $V(r) = 0$ for $r > R$, where V_0 is a positive quantity, solve the Schrödinger equation to obtain the bound states for $l = 0$ and $l = 1$, *i.e.* to find the eigenenergies and the corresponding wavefunctions. You will need spherical Bessel functions, $j_l(kr)$, and spherical Hankel functions, $h_l^{(1)}(i\kappa r)$.
$$j_0(kr) = \frac{\sin kr}{kr}; j_1(kr) = \frac{\sin kr}{(kr)^2} - \frac{\cos kr}{kr}$$
$$h_0^{(1)}(i\kappa r) = -\frac{e^{-\kappa r}}{\kappa r}; h_1^{(1)}(i\kappa r) = i\frac{e^{-\kappa r}}{\kappa r}(1 + \frac{1}{\kappa r})$$

15%

3. Let $\chi_\pm$ be the two spin states and ϕ_{nlm} be the orbital wavefunctions of an electron in a hydrogen atom. Using $\phi_{nlm}\chi_\pm$ as unperturbed wavefunctions and the first order perturbation theory to calculate the energy shifts of the $2p$ states of a hydrogen atom due to the spin-orbit coupling given by
$$H_1 = \frac{e^2}{2m^2c^2}\frac{1}{r^3}\vec{S} \cdot \vec{L}.$$

$$\phi_{21m}(r,\theta,\phi) = \frac{1}{\sqrt{3}}\left(\frac{1}{2a_o}\right)^{1.5}\frac{r}{a_o}e^{-r/2a_o}Y_{1m}(\theta,\phi)$$

10% 4. Describe Hund's rules.

15% 5. Use the orbital wavefunctions and spin states of an electron in the hydrogen-like atom to construct some electronic wavefunctions, $\Psi(\vec{r}_1,\vec{r}_2,\sigma_1,\sigma_2)$'s, with the correct symmetry for the helium atom, where σ_1 and σ_2 are the spin states of the two electrons, *i.e.* χ_+ or χ_-.

15% 6. Prove that if $[H,A]=0$ for the operator A and A does not contain time explicitly, the expectation value of A for any state is a constant of motion.

15% 7. A one-dimensional harmonic oscillator has a mass, m, and a force constant, k. Let $|\,n>$ be the nth lowest eigenstate of this harmonic oscillator. Find the expectation value of x^2 for the state $|\,4>$, *i.e.*

$$<4\,|\,x^2\,|\,4>.$$

You may need $A^\dagger\,|\,n> = \sqrt{(n+1)\hbar}\,|\,n+1>$ and $A\,|\,n> = \sqrt{n\hbar}\,|\,n-1>$, where $A^\dagger$ and A are the raising and lowering operators, respectively.

(一) 利用狹義相對論，計算下列問題

10分 1. 一火箭以 $0.6c$ 的速度離開地球，c 為光速，在地球上一信號站，以每隔6分鐘發出一閃光信號，問在火箭上每隔幾分鐘收到閃光信號？

2. 一質子靜態質量為 m_0，在一巨大電場中加速，若質子受力方向與其運動方向相同，則當質子加速度為 a，速度為 v 時，受力為何？

以上每小題5分

(二) X射線之波長為 λ，撞擊一個以動量 P_0 運動的電子，X射線散射之波長為 λ'，散射角為 ϕ（見圖一），試求 $\lambda' - \lambda$ 之值。

20分 (註：設電子靜態質量為 m_0)

本題20分

圖(一)

(三) 一質點質量為 m，在 $X=0$ 至 $X=a$ 位能井中自由來回運動（見圖二）。在此區域外，質點不出現。

20分 (a) 利用 Wilson-Sommerfeld 量子化規則，求出該質點在基態之能量。

(b) 在基態時該質點測不準動量 (Δp) 與測不準位置 (Δx) 之值，以及動量期望值 $(\bar{p})$ 與質點之位置期望值 $(\bar{x})$ 各若干？

註：$\int x \sin^2 \beta x \, dx = \frac{1}{4}x^2 - \frac{x \sin(2\beta x)}{4\beta} - \frac{\cos(2\beta x)}{8\beta^2}$

以上(a),(b)小題各10分

(四) 在氫原子中電子 spin-orbit interaction energy operator 為

$(\Delta E_{\vec{S}\cdot\vec{L}})_{op} = \frac{1}{2m^2c^2} \cdot \frac{1}{r}\frac{dV}{dr} \vec{S}\cdot\vec{L}$，此式為 c.g.s 制單位，其中 m 為電子質量，

20分 c 為光速，V 為電子位能，$\vec{S}$ 及 $\vec{L}$ 各為電子自旋角動量及軌道角動量。

(a) 用 first order perturbation 理論，求出自旋-軌道交互作用能量。

(b) 計算 $3^2P_{3/2}$ 與 $3^2P_{1/2}$ 能階之差值。（註：r^{-3} 的期望值為 $\overline{r^{-3}} = \frac{1}{a_0^3 n^3 l(l+\frac{1}{2})(l+1)}$，$a_0$ 為 Bohr's radius，n 為主量子數，l 為軌道量子數）

以上每小題10分

科 目：近代物理

(五) 20分 原子 $_{77}Ir^{191}$ first excited state 之生命期為 1.4×10^{-10} sec，高於基態 0.129 MeV，當原子從 first excited state 躍遷至基態時發射 γ-ray. 求(a) first excited state 之能寬 (b) 發射 γ-ray 時，由於原子核的反跳 (recoil) 損失能量若干？若欲使 γ-ray 激發另一相同之 $_{77}Ir^{191}$ 原子核從基態躍遷至 first excited state，產生共振吸收之現象，應如何處理？（planck constant $h=6.63\times10^{-27}$ erg-sec）

以上兩小題，每小題 10分

(六) 10分 一立方晶体晶軸長度為 a，當 X-射線入射於晶体，與晶体之 (111) 平面成 θ 角時，可獲得 first order 極大的繞射紋，試求(a) (111) 平面間之距離 (b) X射線之波長。

以上兩小題，每小題 5分

1. If a force $\mathbf{F}$ is given by $\mathbf{F} = (x^2 + y^2 + z^2)^n (\mathbf{i}\, x + \mathbf{j}\, y + \mathbf{k}\, z)$, find
   (a) $\nabla \cdot \mathbf{F}$, (5%)
   (b) $\nabla \times \mathbf{F}$, (5%)
   (c) A scalar potential $\varphi(x,y,z)$ so that $\mathbf{F} = -\nabla \varphi$. (5%)
   (d) For what value of the exponent n does the scalar potential diverge at both the origin and infinity? (5%)

2. The spherical polar coordinates (r, θ, φ) are defined so that $x = r \sin\theta \cos\varphi$, $y = r \sin\theta \sin\varphi$, and $z = r\cos\theta$.
   (a) Express $\partial/\partial x$, $\partial/\partial y$, $\partial/\partial z$, in spherical polar coordinates. (10%)
   (b) With the quantum mechanical angular momentum operator defined as $\mathbf{L} = -i(\mathbf{r} \times \nabla)$, show that the raising and lowering operators
   $L_x \pm iL_y = \pm e^{\pm i\varphi} (\partial/\partial\theta \pm i \cot\theta\, \partial/\partial\varphi)$. (10%)

3. Consider the vibrations of a classical model of the CO_2 molecule. It is simplified as three masses on the x-axis joined by springs; see Fig.1. The spring forces are assumed to be linear (Hooke's Law) and the mass is constrained to stay on the x-axis. If different coordinates are used for each mass (x_1, x_2, x_3), Newton's second law yields a set of equations. Considering normal modes for the vibrating system,
   (a) write down a matrix-eigenvalue equation corresponding to the Newton's equations, (5%)
   (b) find out the eigenvalues and corresponding eigenvectors. (10%)
   (c) What is the physical meaning for each eigenvalue? (5%)

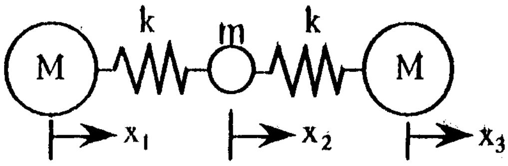


Figure 1.

4. (a) Find a Fourier series to represent the square wave
   $f(x) = 0, \quad -\pi < x < 0,$
   $f(x) = h, \quad 0 < x < \pi.$ (10%)
   (b) Find the Fourier transform of the triangular pulse
   $f(x) = h(1 - a|x|), \quad |x| < 1/a,$
   $f(x) = 0, \quad |x| > 1/a.$ (10%)

5. (a) Show that, for any two operators $\mathbf{A}$ and $\mathbf{B}$,
   $e^{\mathbf{B}}\mathbf{A}e^{-\mathbf{B}} = \mathbf{A} + [\mathbf{B}, \mathbf{A}] + (1/2!)\,[\mathbf{B}, [\mathbf{B}, \mathbf{A}]] + (1/3!)\,[\mathbf{B}, [\mathbf{B}, [\mathbf{B}, \mathbf{A}]]] + \ldots$ (10%)
   (b) The derivative of an operator $\mathbf{A}(\lambda)$ which depends explicitly on a parameter λ is defined to be
   $$d\mathbf{A}(\lambda)/d\lambda = \lim_{\varepsilon \to 0} [\mathbf{A}(\lambda + \varepsilon) - \mathbf{A}(\lambda)]/\varepsilon.$$
   Show that : $d(\mathbf{AB})/d\lambda = (d\mathbf{A}/d\lambda)\, \mathbf{B} + \mathbf{A}\, (d\mathbf{B}/d\lambda)$, and
   $d(\mathbf{A}^{-1})/d\lambda = -\mathbf{A}^{-1} (d\mathbf{A}/d\lambda)\, \mathbf{A}^{-1}$. (10%)